Multiple finances, margins of foreign direct investment and aggregate industry productivity

LEI HOU, PhD candidate
JIARUI ZHANG, PhD candidate

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Lei HOU
PhD candidate at the School of Economics, Jilin University, P.R. China and the Department of Economics, University of Munich, Germany; Ludwigstrasse 28, D-80539, Munich, Germany
e-mail: lei.hou@lrz.uni-muenchen.de

Jiarui ZHANG
PhD candidate at the Department of Economics, University of Munich, Germany; Schackstrasse 4, D-80539, Munich, Germany
e-mail: jiarui.zhang@campus.lmu.de
Abstract
Based on a heterogeneous firm set-up, we model firms’ access to the internal capital market, bank finance as well as bond finance and investigate how firms’ adjustment among multiple sources of finance affects their performance in foreign direct investment and aggregate industry productivity. We find that when facing a bank credit shock (e.g. tighter bank lending), firms with different productivities react differently. Less productive firms exit from the foreign market due to a lack of funds while the more productive resort to bond finance to sustain their multinational status. The increased demand for bond finance as compensation for decreased bank finance by the surviving multinationals exacerbates the competition in the bond market and bids up the bond return rate, which triggers a Melitz-type selection effect through the bond market and brings aggregate industry gains. However, the divestment of those failing FDI firms and the consequently reduced bond financing demand mitigate this effect.

Keywords: bond market, heterogeneous firm, productivity, intensive margin, extensive margin, FDI

1 INTRODUCTION
An emerging body of literature documents the impact of financial development on the facilitation of firm internationalization. While its role in providing a larger scale of external finance and relaxing firms’ financial constraints is widely accepted, it is not clear whether the diversification of financial channels and access to alternative finance accompanied by financial development also play a role. Attention was drawn to the significance of multiple sources of financing by Alan Greenspan after the 1997-98 Asian financial crisis (Greenspan, 2000). He argued that the development of alternative forms of financing helped to fill the funding gap and stabilize business financing, which is especially important when either banks or capital markets freeze up in a crisis. Following this argument and motivated by the credit crunch and simultaneous decline in foreign direct investment (henceforth FDI) in the recent financial crisis, we address the question of whether the availability of alternative financing sources could help reduce the size of the collapse and influence welfare.

Multinational firms have better access to multiple sources of finance than their domestically oriented peers. Firstly, multinational firms are usually large and productive ones (Helpman et al., 2004; Mayer and Ottaviano, 2007). Thus, they have a better chance of accessing market finance other than bank borrowing (Cantillo and Wright, 2000). Moreover, some firms can gain additional financial support from business partners or from the government in the form of trade credit or special policy loans. Secondly, multinational firms have access to finance from different locations. They can obtain finance from their parent country, raise funds from their host country locally or in some cases explore lower-cost finance on a worldwide basis (Antras et al., 2009; Marin and Schnitzer, 2006). Meanwhile, the inter-
nancial capital market among the parent company and its foreign affiliates plays an important role for multinational firms. The allocation of funds through the internal capital market extensively substitutes for external financing when the latter is costly (Desai et al., 2004). Finally, firms tend to keep a precautionary fund reserve to cope with potential risks and uncertainty (Bates et al., 2009; Riddick and Whited, 2009), which is particularly the case for multinational firms, considering the extra cost and higher risk in foreign operations.

Basing on a heterogeneous firm set-up, we model firms’ access to the internal capital market, bank finance as well as bond finance and investigate how firms’ adjustment among multiple sources of finance affects their performance in foreign direct investment and aggregate industry productivity. We find that given exogenous contraction in the supply of bank finance, firms with different productivities react differently. Some less productive firms exit from the foreign market due to less access to bank finance and the high and unaffordable cost of bond finance as a result of tougher competition in the bond market. In comparison, some relatively more productive firms can resort to bond finance as compensation for decreased bank finance to sustain their multinational status. The increased demand for bond finance as a substitute for bank finance by the surviving multinationals exacerbates the competition in the bond market and bids up the bond return rate, which triggers a Melitz-type selection effect through the bond market and brings aggregate industry gains. However, the divestment of those failing FDI firms and thus reduced bond financing demand mitigate this effect.

The contribution of this paper is threefold. Firstly, our paper complements the quickly growing literature on credit constraint and firm internationalization firstly by proposing the impact of alternative financing and differentiating firm responses to the worsening financial condition. Manova (2007) introduces credit constraint into Melitz’s (2003) research and argues that credit constraint restricts firms’ participation and performance in cross-border activity. Arndt et al. (2009), Berman and Hericourt (2008), Buch et al. (2009), Li and Yu (2009) and Muuls (2008) provide supportive evidence for this argument using firm-level data from different countries. We reproduce this result that bad credit conditions impede firms from engaging in FDI. Furthermore, we show that this effect could be mitigated with the existence of alternative financing and could vary across firms with different productivities. Compensation from bond finance and the reallocation of the available funds stabilize firm financing and facilitate FDI. However, only the most productive firms are able to take advantage of multiple sources of finance in smoothing foreign investment.

Secondly, our paper contributes to work on financial systems by analyzing the complementary and substitution effect of bank finance and bond finance. Datta et al. (1999) and Diamond (1991) document the complementary nature of bank finance to bond finance by monitoring. Davis and Mayer (1991) show that the bank
and bond markets can be alternatives to each other but they are not perfect substitutes. Saidenberg and Strahan (1999) focus on the role of bank finance in providing a back-up source and liquidity insurance for bond finance against market shocks. The complementary and substitution effects coexist in our model, which varies across firms. Although the substitution of multiple source of finance could reduce the sensitivity of foreign investment to adverse shocks, only a small fraction of more productive firms benefit from it. The complementary effect of bond finance on bank finance for those less productive firms implies that bond finance cannot fully substitute for bank finance when the banking sector faces a crisis. In our model, it is the higher cost of bond finance over bank finance that hinders less productive firms from employing alternative financing, thus leading to the limited substitutability of bond for bank finance. In reality, the higher cost of bond finance could be the fixed cost of accessing the bond market, or a higher marginal cost for lower-quality firms due to their lower credit rating and higher probability of default. Our result suggests the significance of reducing the cost of bond finance and developing multiple financing sources to satisfy the financing demand of various firms, especially of lower-quality firms.

Thirdly, we propose FDI-induced aggregate productivity gains for the parent country through the selection effect in the capital market. Although the question of whether FDI benefits its host country in productivity through technology spillover to domestic firms is widely discussed (Aitken and Harrison, 1999; Bitzer and Görg, 2005; Haskel et al., 2002; Javorcik, 2004; Keller and Yeaple, 2003), the impact of FDI on the parent country is rarely considered. In contrast to Potelscherghel and Lichtenberg (2001), who present evidence that a country gains from outward FDI through technology sourcing, we show that FDI could bring aggregate productivity gains for the parent country through the reallocation of financial resources towards more productive firms. The tougher competition in the bond market induced by large FDI financing demand selects the least productive firms out of production and enhances aggregate productivity. However, this effect is dampened due to firms’ adjustment among multiple sources of finance.

This paper is organized as follows: section two starts with the model in a closed economy as a benchmark case. After that, we introduce multiple sources of finance in an open economy setting, allowing firms to go abroad where the interaction of bank finance and bond finance and its impact on the margins of FDI are investigated. Section three characterizes the general equilibrium and discusses the aggregate outcome on industry productivity. Section four concludes.

2 MODEL

Consider a world with two countries. We call one country the home (domestic) country and the other the host (foreign) country of FDI. There is a continuum of firms, indexed by \( i \), producing differentiated varieties in each country.
Firm $i$ is born with initial internal fund $N_i$, which is a random number from a common distribution $\Gamma(N_i)$. After paying an entry cost of $f_e$, the firm draws productivity $q_i$ from a common distribution $g(q)$ (Melitz, 2003). With the knowledge of its own productivity, the firm makes the investing decision among three potential options: (1) purchasing corporate bonds $B_i$; (2) investing in domestic production, i.e., producing and selling a distinct product $\omega$ in the home country, the output being denoted by $q_{iD}$; (3) engaging in FDI, i.e., producing and selling $\omega$ in the host country, the output being denoted by $q_{iF}$. Note that the subscript $D$ denotes variables for domestic production whereas $F$ denotes those for foreign production; these apply to the whole paper.

There is a perfect bond market in the economy in which firms can either buy or issue bonds, $B_i$ being positive or negative accordingly. If there is a very low productivity draw, producing is not as profitable as buying bonds. The firm therefore invests all its internal funds in bond holdings to achieve a safe return. If there is a high productivity draw on the contrary, the firm will produce. If its internal fund is not enough to pay the production costs, the firm will raise the working capital by issuing corporate bonds through bond markets.

There is no fixed cost involved in investment in the bond market. In contrast, if the firm engages in production, regardless of whether it is domestic production or FDI, it must pay a fixed overhead cost $f$ to set up the factory. In addition, there is an extra fixed cost $C_F$ for FDI. $f$ and $C_F$ are measured in labor units.

2.1 CLOSED ECONOMY

This subsection provides the closed economy case as a benchmark in which firms only serve the domestic market and obtain external finance merely by issuing corporate bonds.

2.1.1 Demand

The utility function of a representative consumer is

$$U = \left[ \int_{\omega \in \Omega} q(\omega)^{\frac{\varepsilon-1}{\varepsilon}} d\omega \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where the set $\Omega$ represents the mass of available varieties and $\varepsilon$ denotes the elasticity of substitution between any two varieties. Defining the aggregate good $Q=U$ with the aggregate price

$$P = \left[ \int_{\omega \in \Omega} p(\omega)^{1-\varepsilon} d\omega \right]^{\frac{1}{1-\varepsilon}}$$

and solving the expenditure minimization problem of the consumer, we have the demand function for every variety $\omega$. 
2.1.2 Production

Each firm $i$ produces a distinct variety $\omega$ and its output for the domestic market is denoted as $q_{iD}$. Labor is the only input. Define the cost function for producing $q_{iD}$ as:

$$ l_{iD} = \frac{q_{iD}}{\varphi_i} + f $$

(2)

where $f > 0$ is the fixed cost for production, which is the same for any single firm. $\varphi_i$ is the firm-specific productivity. The domestic nominal wage is denoted as $w_{iD}$. Assume that labor must be prepaid.

2.1.3 Bond market

Assume that the bond market is perfect in the sense that it is competitive and there is no information asymmetry, and the equilibrium bond rate is $r$. Firms can invest their internal funds in buying a bond and achieve a return rate of $1 + r$. In comparison, firms whose domestic production is confined by limited internal funds can also issue bonds at the rate of $1 + r$. In the general equilibrium setting, the bond return rate $r$ is determined by the condition that there is no aggregate net demand for bonds. For a single firm, however, $r$ is given.

2.1.4 Firms’ optimal decision

In a closed economy, firm $i$ allocates its own internal fund between bond holding $B_i$ and domestic production $q_{iD}$ (if it produces) and maximizes the total profit from the investment portfolio. Firm $i$ solves

$$ \max_{p_{iD}, B_i} \pi_{iD} = p_{iD} q_{iD} - w_{iD} l_{iD} + r B_i $$

(1)

s.t. $w_{iD} l_{iD} + B_i \leq N_i - f_e$; (2)

where $p_{iD}$ is the product price in the home country. We have:

$$ p_{iD} = \frac{\varphi_i}{\varepsilon - 1} \left( \frac{w_{iD}}{\varepsilon w_{iD} (1 + r)} \right) $$

(3)

$$ q_{iD} = \left[ \frac{(\varepsilon - 1) \varphi_i P}{\varepsilon w_{iD} (1 + r)} \right] Q $$

(4)

$$ l_{iD} = \varphi_i^{-1} \left[ \frac{(\varepsilon - 1) P}{\varepsilon w_{iD} (1 + r)} \right] Q + f $$

(5)

Bond holdings $B_i$ can be calculated from the budget constraint.
Proposition 1 (composition of pricing under limited internal funds): Both the financing cost (bond rate \( r \)) and the labor cost (wage rate \( w_D \) and firm-specific productivity \( \varphi_i \)) compose the product price. Other things being equal, the higher \( r \), higher \( w_D \) or lower \( \varphi_i \), the higher the product price and the lower the output.

In our setting, the derived price \( p_{iD} \) consists of three parts: labor cost \( w_D/\varphi_i \), markup \( \varepsilon/(\varepsilon-1) \) and an additional part \( 1+r \), where \( 1+r \) reflects the extra external financing cost. If a firm does not have sufficient internal funds for production, it issues a bond with a cost of \( 1+r \) to raise working capital. Therefore, the limited internal fund results in a higher price and lower output. Note that to focus the discussion on productivity in this paper, we do not model firm heterogeneity in terms of internal fund \( N \). The effect of \( N \) on firm financing and production works through aggregation. If all the firms have more internal funds (\( N \) increases), they will issue fewer (or hold more) bonds, hence the bond demand increases relative to the supply and the bond return rate \( r \) declines. Other things being equal, the decreased financing cost results in a lower price and the supply of each variety will increase.

2.1.5 Cutoff productivity for domestic production
According to Melitz (2003), a firm’s profit from domestic production depends on its productivity. Less productive firms earn less profit from production and thus have less incentive to produce. Therefore, only those firms with productivities above a certain threshold will produce. What differs in our set-up is that we introduce a perfect bond market where firms can purchase bonds to obtain a safe return. Firms compare the profits from production and those from investing all their internal funds in purchasing bonds and decide to produce if and only if the former is greater than the latter; therefore, the cutoff productivity for domestic production \( \varphi_{iD}^* \) is determined by equation (7) below:

\[
p_{iD}q_{iD} - w_Dl_{iD} + rB_i = r(N_i - f_e)
\]

Using (3), (4), (5) and the binding budget constraint, we have

\[
\varphi_{iD}^* = \left\{ f(\varepsilon - 1) \left[ \frac{\varepsilon w_D (1+r)}{(\varepsilon - 1) P} \right]^\varepsilon Q \right\}^{\frac{1}{\varepsilon-1}}
\]

Proposition 2 (productivity cutoff for domestic production): The productivity cutoff for domestic production \( \varphi_{iD}^* \) is higher with a higher fixed production cost \( f \), higher labor wage \( w_D \) or higher financing cost \( r \).
Intuitively, the above increasing relationship means a higher cost (either a real or financial cost), which requires higher productivity for firms to engage in production. The shape of the increasing relationship depends on $\varepsilon$. For example, when $\varepsilon$ is less than 2, the cutoff productivity is convex in $f$; while when $\varepsilon$ is larger than 2, it is concave in $f$. As for the impact of the firm’s internal fund, it only works through a bond market in aggregation. As we discussed in proposition 1, firms’ bond holding increases with their internal funds. More aggregate internal funds could pull down the bond rate and result in a lower cutoff productivity. However, in partial equilibrium, the bond rate is exogenous for a single firm. Therefore, the internal fund is not directly related to the firm-level cutoff productivity.

2.2 OPEN ECONOMY

In this subsection, we consider the case of an open economy in the sense that firms are interested in producing domestically as well as expanding production to a foreign country by means of FDI. Meanwhile, we introduce going-abroad-oriented bank credit as alternative financing and reconsider the above firm’s investment portfolio decision. The cutoff productivity for a firm to become a multinational is also derived. Moreover, the interaction of borrowing from a bank and issuing corporate bonds and the overall effect of multiple sources of finance are discussed.

2.2.1 Demand

For simplicity and without loss of generality, we assume the aggregate price index and aggregate goods index in the host country are the same as those in the home country, and are denoted again as $P$ and $Q$, respectively. We further impose the assumption that when the economy shifts from autarky to openness, $P$ and $Q$ will not change. In other words, the new varieties coming in as the result of openness will not affect the aggregate indices. The demand function for each variety in the host country is given by:

$$q_{iF} = \left( \frac{P}{P_{iF}} \right)^\varepsilon Q$$

(9)

2.2.2 Production

Assume firm $i$’s productivity spills over to its foreign affiliate and it produces in the foreign country with the same productivity as in the home country while it has to shoulder an extra fixed cost $C_F$ to carry out FDI. This foreign expansion-induced fixed cost includes the expenses for building up foreign affiliates and distribution channels, collecting information about the foreign market and foreign regulations, etc. Regardless of the form of such a cost, it is independent of the firm’s output and must be paid before the firm’s revenue in the foreign market is fulfilled. This cost $C_F$ is assumed to be uncertain for the firm at the moment when a firm arranges its investment portfolio. The distribution of $C_F$ is common knowledge.
and the FDI decision is made based on the firm’s expectation for \( C_F \). \( C_F \) is revealed when the firm sets foot on the foreign land. FDI is successful (hence FDI profit is received) only if \( C_F \) is fully covered.

In an open economy, the domestic production function is the same as equation (2), whereas the production function for FDI is given as:

\[
q_{IF} = \frac{q_{IF}}{\varphi_i} + f + C_F
\]

where \( q_{IF} \) and \( l_{IF} \) are respectively output and labor input in the foreign country. Here assume that the extra fixed cost \( C_F \) follows a concave distribution \( f(C_F) \) with support \([0, \infty]\). The \( f(C_F) \) has the cumulative distribution \( F(C_F) \).

### 2.2.3 Going-abroad-oriented loans and the probability of foreign direct investment success

To cover \( C_F \), the firm can obtain finance from banks. Assume that a going-abroad-oriented bank loan is available for all FDI firms. Such loans aim to release firms’ financial constraints due to the substantial upfront costs of FDI and are therefore assumed to be used only to shoulder \( C_F \).\(^1\) Collateral is required by banks. Firm \( i \) pledges a fraction \( \tau \), \( \tau \in (0,1] \) of the overhead fixed cost \( f \) as collateral to obtain a bank loan of the amount of \( \mu \tau f \), where \( \mu \) is the multiplier over the collateral. Here we use \( \mu \) to measure the availability of external bank credit, which is an indicator of country-specific financial development. The higher \( \mu \) implies better access to external credit and better financial development of a country. We further assume that borrowing from banks is costless as bankers are competitive and have no access to the bond market.

Moreover, to guarantee the sufficiency of funds to cover \( C_F \) and thus the success of FDI, firms may keep some reserve funds \( A \), together with the bank borrowing \( \mu \tau f \), to pay the extra fixed cost. \( A \) could be a fraction of the internal fund or financed from the bond market. Therefore, before \( C_F \) is revealed, the firm has \( A + \mu \tau f \) prepared. Hence, the probability of the FDI’s success is \( \text{Prob} (C_F \leq A + \mu \tau f) \), which is \( F(A + \mu \tau f) \) and it is endogenized. As we shall see, for FDI firms, the more productive the firm is, the larger \( A \) is kept and the more likely that the FDI will be successful.\(^2\)

### 2.2.4 Firms’ optimal decision

Firm \( i \) maximizes the expected total profit from bond holding, domestic production and FDI.

\(^1\) By this assumption, we rule out the case of firms using this loan to pay for domestic production so that we can obtain results in an open economy that are comparable to those in a closed economy and focus on the effect of the bank loan on firms’ financing strategy and FDI decisions.

\(^2\) We try to model the reality that productive multinational firms issue corporate bonds to raise working capital for FDI since the profits from FDI are sufficiently large to cover the financing cost.
\[
\max_{p_{ID}, p_{IF}, A_i, B_i} \quad E[\pi_i] = E \left\{ p_{ID} q_{ID} - w_{D} l_{ID} + \left( p_{IF} q_{IF} - w_{F} l_{IF} \right) F \left( A_i + \mu \tau f \right) + r B_i \right\}
\]

s.t. \quad w_{D} l_{ID} + w_{F} \left( l_{IF} - C_F \right) + A_i + B_i \leq N_i - f_e \quad \text{for} \quad (1); (2); (9); (10)

Note that the profit from FDI is multiplied by the probability of its success. Also note that in the budget constraint, \( C_F \) is covered by \( A \) and \( \mu \tau f \). Denoting the expected value of \( C_F \) as \( C_r \), we have:

\[
p_{ID} = \frac{\epsilon}{\epsilon - 1} \frac{w_D}{\varphi_i} (1 + r) \quad (11)
\]

\[
p_{IF} = \frac{\epsilon}{\epsilon - 1} \frac{w_F}{\varphi_i} \left( 1 + \frac{r}{F(A_i + \mu \tau f)} \right) \quad (12)
\]

\[
B_i = N_i - f_e - w_{D} l_{ID} - w_{F} \left( l_{IF} - C \right) - A_i \quad (13)
\]

and \( A_i \) is determined by:

\[
\left( p_{IF} q_{IF} - w_{F} l_{IF} \right) f \left( A_i + \mu \tau f \right) = r \quad (14)
\]

Equations (11)–(14) characterize the optimal choices of an FDI firm. We can compare the prices in the home country and the host country by comparing (11) and (12), noticing that \( F(A + \mu \tau f) \leq 1 \).

The price for the domestic market has the same expression as that in the closed economy benchmark (equation (3) in section 2.1.4), which means that firms do not change their pricing strategy for the home market when they start foreign business. Nevertheless, the actual value of the domestic price may be different. When the economy shifts from autarky to openness, firms of high productivity adjust their investment portfolios: they purchase fewer bonds (or issue more bonds) and allocate funds to FDI. The adjustment, as will be discussed in aggregation in section 3, induces tougher competition in the bond market and drives the bond return rate up. Hence, the actual price in the home market under an open economy setting will be higher than in a closed economy, although they share the same mathematical expression.

An implicit solution of firm \( i \)'s choice of \( A \) is given by equation (14). The simulation results are provided in appendix A (where propositions 3 to 6 are simulated). We have the following proposition:

**Proposition 3 (reserve fund for FDI):** The reserve fund for FDI \( A_i \) is higher with higher productivity \( \varphi_i \), lower credit access \( \mu \), lower production fixed cost \( f \) or lower bond financing cost \( r \).
The relationship between \( A \) and \( \mu \) suggests a firm’s substitution in multiple sources of finance. When bank credit is tighter, a firm increases \( A_i \) as the alternative source to cover \( C_f \), so that it can maintain FDI. This finding supplements the existing literature in which firms are left helpless and exit production when bank credit is tight (Buch et al., 2009; Manova, 2007). In our model, however, firms can resort to alternative finance and keep production unaffected.

Note that borrowing from a bank has no cost but \( A_i \) has a cost of \((1 + r)\), because \( A_i \) is raised either from internal funds or from the bond market. If the bond return rate is higher, it is more attractive to buy bonds than to produce, hence the firm will cut \( A_i \).

As for the negative role of fixed cost \( f \) in \( A \), it works in two ways. On one hand, \( f \) is a real cost of FDI. The higher the cost is, the less incentive there is for firms to undertake FDI, and hence the smaller the reserve fund firms keep for FDI projects. On the other hand, \( f \) could be used as collateral: firms can obtain greater bank loans against a larger \( f \), so they could reduce the amount of the reserve fund.

An important finding is that more productive firms keep more reserve funds and thus have a higher probability of success in producing abroad. As FDI is more profitable with higher productivity, those firms have incentives to guarantee the FDI’s success. This result differs from the previous literature, in which the probability of success or the probability of firms’ default is assumed to be exogenous and independent of firm productivity (e.g. Buch et al., 2009; Manova, 2007). In Li and Yu (2009), more productive firms have a higher probability of success but such a relationship is \textit{ex ante} given without a micro foundation. In our model, however, the probability is firm-specific and firms themselves choose how much to “invest” to increase the probability of success.

**Proposition 4 (intensive margin of FDI):** The more productive a firm is (higher \( \varphi \)), the larger is its affiliate sale. The sale is also larger if the wage cost \( w_F \) is lower or the bond financing cost \( r \) is lower. If a firm can maintain FDI after a credit crunch\(^3\) (decrease in credit multiplier \( \mu \)), it raises working capital from issuing bonds and keeps its affiliate sale unaffected.

The first three arguments on \( \varphi, w_F \) and \( r \) are intuitive and easily verified through equation (12). Higher productivity or a lower cost, either the wage cost or the financial cost, results in more output and sales. However, the change in bank credit availability \( \mu \) triggers firms’ adjustment to their financing strategy and affects affiliate sales indirectly. In partial equilibrium, when bank credit suddenly becomes tight, firms raise more funds from the bond market to substitute for bank credit in order to keep their working capital. In our model, when \( \mu \) decreases to such an extent that there is less borrowing, if a firm can maintain FDI, it will increase \( A_i \) (proposition 3) to keep the probability of the FDI’s success. Therefore,

\(^3\) We will discuss the condition for firms to maintain FDI in section 2.2.5.
according to equation (12), as long as the bond return rate does not change in partial equilibrium, the affiliate sale $q_{iF}$ will not be affected. This result is consistent with the evidence that during the recent financial crisis, a non-negligible fraction of firms reallocated more funds to finance working capital and their sales remain unchanged or even expand, especially in domestic-oriented or non-tradable sectors (World Bank Financial Crisis Survey 2010; 2010 Survey on Current Conditions and Intention of Outbound Investment by Chinese Enterprises).

### 2.2.5 Cutoff productivity for foreign direct investment

Following the argument in section 2.1.5, the cutoff productivity for FDI is derived by equation (15), the LHS of which is the profit when the firm engages in domestic production as well as FDI while the RHS is the profit when the firm merely serves the domestic market. The firm will expand production to the foreign country if and only if its total profit is higher than that from only serving the domestic market.

$$p_{ID}q_{ID} - w_{D}l_{ID} + \left(p_{IF}q_{IF} - w_{F}l_{IF}\right)F\left(A + \mu \tau f\right) + rB_{IF} = p_{ID}q_{ID} - w_{D}l_{ID} + rB_{ID}$$  \hspace{1cm} (15)

where $B_{IF}$ comes from (13) and $B_{ID}$ comes from (6). Then we derive the expression of cutoff productivity for FDI:

$$\phi_{IF}^* = \frac{f\left(1 + F\right)}{Q}\left[\frac{\epsilon w_{F}\left(1 + \frac{r}{F}\right)}{(\epsilon - 1)P}\right]^{1/\epsilon}\frac{1}{1 + \frac{FC + rA}{(F + r)f}}$$  \hspace{1cm} (16)

where $F$ denotes $F(A + \mu \tau f)$.

**Proposition 5 (extensive margin of FDI):** The productivity cutoff for FDI $\phi_{IF}^*$ is lower when firms have better access to credit (higher credit multiplier $\mu$), lower bond financing cost $r$, lower production fixed cost $f$ or $C$, and lower labor wage $w_F$. The expected profit of undertaking FDI is larger with a higher $\mu$.

With the support of better availability of bank credit, more firms are able to go abroad. Meanwhile, the induced higher expected profits make FDI more attractive to firms. This result implies better credit conditions as a result of the financial development in a country, which plays a positive role in facilitating firm internationalization. On the contrary, various costs, such as the labor wage, overhead cost and financial cost, impede firms going abroad.

**Proposition 6 (cutoff gap):** The gap between the productivity cutoff for FDI and the cutoff for domestic production ($\phi_{IF}^* - \phi_{ID}^*$) is lower facing lower bond rate $r$, larger credit multiplier $\mu$ and lower expected fixed cost $C$.
Comparing equation (16) with equation (8), and knowing that $F(A + \mu_1) \leq 1$, we immediately conclude that $\phi_{i,F}^* > \phi_{i,D}^*$. Due to the existence of extra fixed costs, firms require higher productivity to attain positive profits from FDI. The two cutoffs are equal if and only if $C = 0$. In this case, the probability of a successful FDI is 1 and firms will not keep any $A$ as it is not necessary and $A$ is costly. Proposition 6 states that a better credit condition (higher $\mu$) or lower bond financing cost (lower $r$) can reduce the productivity requirement for FDI and promote domestic firms’ growth into multinationals. Note that when facing a lower bond return rate $r$, both cutoffs decrease, while that for FDI declines faster, indicating the higher sensitivity of FDI to financing conditions compared with domestic production.

2.2.6 Complementarity and substitution of multiple sources of finance

FDI firms have access to two external sources of finance, i.e. borrowing from banks and issuing corporate bonds. When facing a bank credit shock, firms adjust their financing strategy and fund allocation among investment projects but firms with different productivities react differently. Take a bad credit shock as an example.

When bank credit suddenly becomes tight, i.e. $\mu$ suddenly decreases, $\phi_{i,F}^*$ increases (proposition 5) and some relatively less productive FDI firms are forced to exit from FDI. As a result of withdrawing capital from FDI, these firms issue fewer bonds. In this case, deteriorative bank credit results in shrinking bond issuance, which we call the complemental effect of bond issuing and bank borrowing.

In contrast, however, those firms that are productive enough to maintain FDI under a worse credit condition issue more bonds as a substitution for reduced bank credit and keep the working capital for foreign production unchanged, which we call the substitution effect of bond issuing and bank borrowing. Because of the possibility of issuing bonds as an alternative form of finance, firms do not necessarily experience production contraction when facing credit tightness, which implies the significance of multiple sources of finance in smoothing investment.

Figure 1 intuitively depicts the change in $A$ with decreased $\mu$ (from $\mu$ to $\mu'$) and the differentiation of firms in financing. As we mentioned above, only those firms with productivities that are higher than the cutoff productivity for FDI keep reserve fund $A$. The more productive the firm is, the more $A$ it raises (proposition 3). Therefore, $A$ is 0 for firms with productivities lower than $\phi_{i,F}^* (\mu)$, and $A$ jumps to positive at the cutoff value $\phi_{i,F}^* (\mu)$ and keeps increasing with $\phi$ after that.

Facing a bank credit shock ($\mu$ decreases to $\mu'$), the cutoff of carrying out FDI increases from $\phi_{i,F}^* (\mu)$ to $\phi_{i,F}^* (\mu')$. The firms with productivities in between exit from FDI and hence do not reserve $A$ any longer, while those firms with productivity higher than $\phi_{i,F}^* (\mu')$ maintain FDI and raise more $A$ from issuing bonds. As the adjustment of $A$ responding to the alteration of the bank credit condition is through bond finance, figure 1 shows the complementary and substitution effect of bond finance and bank finance.
3 AGGREGATION

3.1 CHARACTERIZATION OF EQUILIBRIUM IN AN OPEN ECONOMY

In an open economy, stationary general equilibrium is characterized as follows: 
(1) there is an aggregate cutoff productivity for domestic production $\varphi^*_D$, which is determined by equalizing the profit from purely holding bonds and that from producing domestically; (2) there is an aggregate cutoff productivity for FDI $\varphi^*_F$, which is determined by comparing the total profit from engaging in domestic production and FDI with that from merely domestic production; (3) a mass $M$ of incumbent firms is partitioned into three groups in terms of productivity. Firms with productivity higher than $\varphi^*_F$ produce domestically as well as abroad. Firms with productivity lower than $\varphi^*_D$ do not produce but invest in purchasing bonds. Firms with productivity in between produce and serve the domestic market; (4) a firm’s entry decision is made by equalizing the present value of the expected average profit flows $\bar{\pi}$ of all types of firms and the sunk cost for entry $f_e$; (5) in each period, a mass $M_e$ of new entrants replaces the mass of $\delta M$ of incumbent firms that are hit by a bad shock with the probability of $\delta$ and exit; (6) product markets clear such that the consumers’ demand is met by the firms’ supply; (7) the labor market clears to determine the wage $w$ (we assume the inelastic supply of labor $L$); (8) the bond market clears in the sense that there is no aggregate net demand for bonds, where the bond rate $r$ is determined; (9) the resource constraint is satisfied such that the total income equals the total expenditure. The derivation of the general equilibrium is given in appendix B.

3.2 AGGREGATE OUTCOME OF THE COMPLEMENTARY EFFECT AND SUBSTITUTION EFFECT

As we discussed above, when an adverse shock on bank credit occurs, the complementary effect implies that FDI divestment and the purchase of more bonds, whe-
the substitution effect means that firms issue more bonds to finance FDI. In general equilibrium, the complementary effect and substitution effect influence the demand and supply in the bond market and thus the bond rate oppositely. The overall outcome is a result of the relative scale of the two effects, which further relies on the distribution of firm productivity and the severity of shocks on bank credit. In a country where the firm distribution skews towards high productivity, more firms will sustain FDI and the substitution effect will be dominant. As a result, the bond rate will increase. The effect of shocks is more complicated. On one hand, when facing a more severe adverse shock, more firms exit from FDI and transfer capital to purchasing bonds. On the other hand, the survivors in FDI will issue more bonds to compensate for the reduced bank finance. Consequently, both the complementary effect and the substitution effect are stronger and the overall effect is ambiguous.

3.3 SELECTION EFFECT IN THE BOND MARKET AND AGGREGATE INDUSTRY PRODUCTIVITY

When an economy opens, those productive firms that go abroad will issue more bonds from the parent country to finance foreign production. The increased supply in the bond market will bid up the bond return rate and thus increase the financing cost for all the producing firms, either FDI or non-FDI firms. Facing a higher financing cost, the least productive producing firms are forced to exit from production and switch to being purely bond holders. Thus, the aggregate productivity of the producing firms increases. Therefore, outward FDI triggers the selection effect through the bond market and brings aggregate industry productivity gains for the parent country.

As previously discussed in section 3.2, a shock to the bank credit supply can also influence the bond return rate and hence further the aggregate productivity gains. However, whether the change in bank credit conditions will intensify or weaken such gains depends on the relative importance of the above complementary effect and substitution effect. As a response to an adverse shock to bank finance, the rising bond rate as a result of the substitution effect will shuffle the deck and wash out less productive firms. However, the existence of the complementary effect pulls down the bond rate and mitigates this selection.

4 CONCLUSION

This paper introduces the internal fund, bank finance and bond finance into a heterogeneous firm set-up and analyzes firms’ adjustment among multiple sources of finance and its impact on the performance of FDI and aggregate industry productivity. We show that with access to the bond market as an alternative source of financing, firms suffering from bank lending tightness could stabilize their financing and maintain FDI. However, only a proportion of more productive firms benefit from the substitution of bond finance for bank finance. In comparison, the less efficient firms could not afford the higher cost of bond finance due to the in-
creased competition in the bond market, and thus exit from production. Therefore, the rising bond rate induces the reallocation of financial resources from less efficient firms towards more efficient ones and thus increases the aggregate industry productivity of the producing firms. Nevertheless, the decreased financing demand of divesting firms helps to pull down the bond rate and thus weakens the above effect.

Our results suggest the significance of the diversification of financial channels and the availability of alternative financing in smoothing foreign investment, which is particularly important for low-quality firms. Moreover, the selection through the bond market implies the role of the capital market in reshuffling firms, which also proposes a mechanism of FDI-induced welfare for parent countries.

To focus on the role of alternative financing in stabilizing investment, we did not discuss the difference between bank finance and bond finance in this paper. However, modeling their differences in restructuring, monitoring and screening will help us to understand better the limited substitutability of the two sources of finance and might generate more fruitful results. Moreover, modeling the financing sources of bank sectors and investigating the co-movement of the bank sector and the bond market constitute another direction for future research. In addition, relaxing the perfect competition assumption for the bond market and introducing a firm-specific bond rate are also interesting extensions.
APPENDIX A

Numerical simulations of the propositions

Propositions 1 and 2 are straightforward, so here we only provide the simulation results for propositions 3, 4, 5 and 6.

**Distribution of the fixed cost for FDI:** Assume $C_F$ follows Pareto distribution

$$F(x) = \Pr(C_F \leq x) = 1 - \left(\frac{b}{x}\right)^k$$  \hspace{1cm} (A.1)

with the support of $[b, \infty]$, where $b$ and $k$ are parameters of the distribution. The probability density function of $C_F$ is therefore given by

$$f(x) = \frac{kb^k}{x^{k+1}} = \frac{k}{x} \left(\frac{b}{x}\right)^k$$  \hspace{1cm} (A.2)

Denote the mean of $C_F$ as $c$, then $c = E(C_F) = \frac{kb}{k-1}$.

**A.1 SIMULATION OF PROPOSITION 3**

The optimal reserve fund $A$ for an FDI firm is given by equation (14)

$$\left(p_{iF} q_{iF} - w_{F} l_{iF}\right) f\left(A_i + \mu \tau f\right) = r.$$  \hspace{1cm} (A.4)

By inserting equations (9), (10), (12), (A.1) and (A.2) into equation (14) we obtain equation A.3 for the simulation.

$$\left[\frac{\varepsilon - \frac{w_F}{\varepsilon - 1 \varphi_i} \left(1 + \frac{r}{\left(1 - \frac{b}{A_i + \mu \tau f}\right)^k}\right)}{1 - \left(\frac{b}{A_i + \mu \tau f}\right)^k}\right]^{1-\varepsilon} P^{\varepsilon Q} \left[\frac{1 - \frac{-1}{\varepsilon} \frac{1}{1 + \frac{r}{\left(1 - \frac{b}{A_i + \mu \tau f}\right)^k}}}{}\right]$$

$$- w_F (f + c) \left(\frac{b}{A_i + \mu \tau f}\right)^k = r$$  \hspace{1cm} (A.3)

Figures A.1.1–A.1.4 depict the change in $A$ with $\mu$, $r$, $f$ and $\varphi$, respectively.
**Parameter values**

Figure A.1.1: $r = 0.05$, $\varphi_i = 0.5$, $f = 10$, $\varepsilon = 2$, $w_F = 1$, $P = 10$, $Q = 10$, $\tau = 0.5$, $b = k = 3$, $c = 4.5$

Figure A.1.2: $\mu = 1.4$, $\varphi_i = 0.5$, $f = 10$, $\varepsilon = 2$, $w_F = 1$, $P = 10$, $Q = 10$, $\tau = 0.5$, $b = k = 3$, $c = 4.5$

Figure A.1.3: $r = 0.05$, $\mu = 1.4$, $\varphi_i = 0.5$, $\varepsilon = 2$, $w_F = 1$, $P = 10$, $Q = 10$, $\tau = 0.5$, $b = k = 3$, $c = 4.5$

Figure A.1.4: $r = 0.05$, $\mu = 1.4$, $f = 10$, $\varepsilon = 2$, $w_F = 1$, $P = 10$, $Q = 10$, $\tau = 0.5$, $b = k = 3$, $c = 4.5$

**A.2 SIMULATION OF PROPOSITION 4**

We derive the solution for $q_{iF}$ (A.4) by inserting equation (12) and the distribution of $C_F$ into equation (9), where variable $A$ is determined by equation (A.3).

$$q_{iF} = P^A Q \left\{ \frac{\varepsilon w_F}{\varepsilon - 1} \varphi_i \left[ 1 + \frac{r}{b} \left( \frac{1}{A + \mu \tau f} \right)^k \right] \right\}^{-\varepsilon} \quad (A.4)$$
Figures A.2.1–A.2.4 show the change in $q_{iF}$ with $r$, $\omega$, $\varphi$ and $\mu$, respectively.

**Parameter values**

Figure A.2.1: $\mu = 1.4$, $\varphi_i = 0.5$, $w_F = 1$, $\varepsilon = 2$, $f = 10$, $P = 10$, $Q = 10$, $\tau = 0.5$, $b = k = 3$, $c = 4.5$

Figure A.2.2: $r = 0.05$, $\mu = 1.4$, $\varphi_i = 0.5$, $\varepsilon = 2$, $f = 10$, $P = 10$, $Q = 10$, $\tau = 0.5$, $b = k = 3$, $c = 4.5$

Figure A.2.3: $r = 0.05$, $\mu = 1.4$, $w_F = 1$, $\varepsilon = 2$, $f = 10$, $P = 10$, $Q = 10$, $\tau = 0.5$, $b = k = 3$, $c = 4.5$

Figure A.2.4: $r = 0.05$, $\varphi_i = 0.5$, $w_F = 1$, $\varepsilon = 2$, $f = 10$, $P = 10$, $Q = 10$, $\tau = 0.5$, $b = k = 3$, $c = 4.5$

**A.3 SIMULATION OF PROPOSITION 5**

Inserting $F = 1 - \left( \frac{b}{A + \mu \tau f} \right)^k$ into equation (16), we obtain equation (A.5) for simulation relating to $\varphi^*_F$. Variable $A$ in equation (A.5) is determined by equation (A.3) in which $\varphi_i$ takes the value of $\varphi^*_F$. 
The total profit of FDI firms is:
\[
\pi_i = p_{iD}q_{iD} - w_{DF}l_{iD} + (p_{iF}q_{iF} - w_{F}l_{iF})F_i (A_i + \mu \tau f) + rB_i \tag{A.6}
\]

Inserting the optimal solutions of firms’ profit maximization problem given by equations (1), (2), (9), (10), (11), (12) and (13) into (A.6) and rearranging, we obtain the final simulation equation for \( \pi \).

Variable \( A \) in equation (A.7) is determined by equation (A.3).

Figures A.3.1–A.3.4 show the change of \( \varphi_{iF}^* \) with \( \mu, r, f \) and \( w_{F} \), respectively. Figure A.3.5 depicts the increasing relationship of \( \pi \) with \( \mu \).
Parameter values

Figure A.3.1: $r = 0.05$, $f = 10$, $w_F = 1$, $\varepsilon = 2$, $\varphi_i = 0.5$, $P = 10$, $Q = 10$, $\tau = 0.5$, $b = k = 3$, $c = 4.5$

Figure A.3.2: $\mu = 1.4$, $f = 10$, $w_F = 1$, $\varepsilon = 2$, $\varphi_i = 0.5$, $P = 10$, $Q = 10$, $\tau = 0.5$, $b = k = 3$, $c = 4.5$

Figure A.3.3: $r = 0.05$, $\mu = 1.4$, $w_F = 1$, $\varepsilon = 2$, $\varphi_i = 0.5$, $P = 10$, $Q = 10$, $\tau = 0.5$, $b = k = 3$, $c = 4.5$

Figure A.3.4: $r = 0.05$, $f = 10$, $\mu = 1.4$, $\varepsilon = 2$, $\varphi_i = 0.5$, $P = 10$, $Q = 10$, $\tau = 0.5$, $b = k = 3$, $c = 4.5$

Figure A.3.5: $r = 0.05$, $f = 10$, $\varepsilon = 2$, $\varphi_i = 0.5$, $P = 10$, $Q = 10$, $\tau = 0.5$, $b = k = 3$, $c = 4.5$, $w_D = w_F = 1$, $f_F = 10$, $N = 500$
The simulation equation for $\varphi^*_F - \varphi^*_D$ is derived by equation (A.5) minus equation (8).

Figures A.4.1–A.4.3 describe the change in the cutoff gap $\varphi^*_F - \varphi^*_D$ with $r$, $\mu$ and $c$, respectively. Note that given $k$, the relationship of $\varphi^*_F - \varphi^*_D$ and $c$ is indirectly represented by the change in $\varphi^*_F - \varphi^*_D$ with $b$.

**Parameter values**

- **Figure A.4.1**: $\mu = 1.4$, $f = 10$, $\varepsilon = 2$, $\varphi_i = 0.5$, $P = 10$, $Q = 10$, $\tau = 0.5$, $b = k = 3$, $c = 4.5$, $w_D = w_F = 1$
- **Figure A.4.2**: $r = 0.05$, $f = 10$, $\varepsilon = 2$, $\varphi_i = 0.5$, $P = 10$, $Q = 10$, $\tau = 0.5$, $b = k = 3$, $c = 4.5$, $w_D = w_F = 1$
- **Figure A.4.3**: $r = 0.05$, $\mu = 1.4$, $\varepsilon = 2$, $\varphi_i = 0.5$, $P = 10$, $Q = 10$, $\tau = 0.5$, $k = 3$, $w_D = w_F = 1$
APPENDIX B

Sketch of the general equilibrium in an open economy

Following Melitz (2003), we assume that there is an unlimited number of prospective firms waiting to enter our model. Each firm was born with an initial fund $N$. To enter, they first have to pay entry cost $f_e$ with their initial fund to draw their own productivities from a common distribution $g(\varphi)$. $g(\varphi)$ is Pareto distribution with cumulative density function $G(\varphi)$ and the support of $[b, \infty]$ (Helpman et al., 2004). Firms with high productivity produce, among which the higher ones also engage in FDI, while those with low productivity hold bonds only. All the firms face a constant probability $\delta$ of forced exit in each period. The forced exit firms can pay $f_e$ to draw new productivity again.

Denotations of endogenous variables: $M$ number of incumbent firms; $M_e$ number of new entrants in each period; $\bar{\pi}$ average profit across all types of firms; $\varphi_D^*$ cutoff productivity for domestic production; $\varphi_F^*$ cutoff productivity for FDI; $P$ price index; $Q$ aggregate goods; $w$ wage; $r$ bond rate.

The steady-state equilibrium is characterized by the following equations.

Zero cutoff profit for domestic production:

$$\varphi_D^* = \left[ \frac{f (\varepsilon - 1) \left[ \frac{\varepsilon w (1 + r) \varphi}{(\varepsilon - 1) P} \right]^{1/\varepsilon}}{Q} \right]^{1/\varepsilon - 1}$$  \hfill (B.1)

Zero cutoff profit for FDI:

$$\varphi_F^* = \left[ \frac{f (\varepsilon - 1) \left[ \frac{\varepsilon w (1 + r / F) \varphi}{(\varepsilon - 1) P} \right]^{1/\varepsilon}}{Q} \left[ 1 + \frac{FC + r A / w}{(F + r) f} \right] \right]^{1/\varepsilon - 1}$$  \hfill (B.2)

Expected average profit:

$$\bar{\pi} = G(\varphi_D^*) r (N - f_e) + \int_{\varphi_D^*}^{\varphi_F^*} \pi_D u_D(\varphi) d\varphi + \left[ 1 - G(\varphi_F^*) \right] \int_{\varphi_F^*}^{\infty} \pi_F u_F(\varphi) d\varphi$$  \hfill (B.3)

where: $\pi_D = p_D q_D - w l_D + r B_D$, and $\pi_F = p_D q_F - w l_D + (p_F q_F - w l_F) F (A + \mu f') + r B_F$. 
Free entry condition: \[
\frac{\bar{\pi}}{\delta} = f_e \quad \text{(B.4)}
\]

Firm entry equals firm exit: \[
M_e = \delta M \quad \text{(B.5)}
\]

Labor market clearing condition:
\[
L = M \left[ G(\varphi_F^*) - G(\varphi_D) \right] \int_{\varphi_D}^{\varphi_F} l_D(P) d\varphi + M \left[ 1 - G(\varphi_F^*) \right] \int_{\varphi_F}^{\infty} l_D(P) d\varphi
+ M \left[ 1 - G(\varphi_F^*) \right] \int_{\varphi_F}^{\infty} l_F(P) d\varphi
\quad \text{(B.6)}
\]

where \( L \) is the exogenous total supply of the economy, \( l_D = \varphi^{-1} \left[ \frac{(\varepsilon-1)P}{\varepsilon W(1+r)} \right]^\varepsilon Q + f \)

and \( l_F = \varphi^{-1} \left[ \frac{(\varepsilon-1)P}{\varepsilon W(1+r)} \right]^\varepsilon Q + f + c \).

Bond market clearing condition:
\[
MG(\varphi_D)(N - f_e) + M \left[ G(\varphi_F^*) - G(\varphi_D) \right] \int_{\varphi_D}^{\varphi_F} B_D u_D(\varphi) d\varphi
+ M \left[ 1 - G(\varphi_F^*) \right] \int_{\varphi_F}^{\infty} B_F u_F(\varphi) d\varphi = 0 \quad \text{(B.7)}
\]

where \( B_D = N - f_e - w l_D \) and \( B_F = N - f_e - w l_D - w(l_F - c) - A \).

Price index:
\[
P = \left[ M \left[ G(\varphi_F^*) - G(\varphi_D) \right] \int_{\varphi_D}^{\varphi_F} p_D^{1-\varepsilon} u_D(\varphi) d\varphi + M \left[ 1 - G(\varphi_F^*) \right] \int_{\varphi_F}^{\infty} p_D^{1-\varepsilon} u_F(\varphi) d\varphi
+ M \left[ 1 - G(\varphi_F^*) \right] \int_{\varphi_F}^{\infty} p_F^{1-\varepsilon} u_F(\varphi) d\varphi \right]^{1/(1-\varepsilon)} \quad \text{(B.8)}
\]

Resource constraint:
\[
(N - f_e) M_e + PQ = w L \quad \text{(B.9)}
\]

In equation (B.1) – (B.9),
\[
u_D(\varphi) = \begin{cases} 
\frac{g(\varphi)}{G(\varphi_F^*) - G(\varphi_D)} & \text{if } \varphi \geq \varphi_F^* \\
0 & \text{if } \varphi < \varphi_F^*
\end{cases}
\]
$$u_F(\varphi) = \frac{g(\varphi)}{1 - G(\varphi^*_F)} \text{ if } \varphi \geq \varphi^*_F \text{ and } u_F(\varphi) = 0 \text{ if } \varphi < \varphi^*_F.$$ 

$$G(\varphi) = 1 - \left( \frac{b}{\varphi} \right)^k, \quad g(\varphi) = \frac{k}{\varphi} \left( \frac{b}{\varphi} \right)^k$$

and $A$ is an implicit function of $\varphi$, which is determined by equation (14) 

$$\left( p_{iF} q_{iF} - w_F l_{iF} \right) f \left( A_i + \mu \tau f \right) = r.$$
LITERATURE


